# Models and Algorithms for Passenger Railway Optimization Planning Problems

Passenger railway systems are highly complex systems requiring the solution of several planning problems that can be analyzed and solved through the application of mathematical models and optimization techniques, which generally lead to an improvement in the performance of the system, and also to a reduction in the time required for solving these problems.

The planning process is generally divided into several phases: *Line Planning*, *Train Timetabling*, *Train Platforming*, *Rolling Stock Circulation* and *Crew Planning*. In this lecture, after a description of the whole planning process and of its main phases, the *Train-Unit Assignment Problem*, an important NP-hard problem arising in the planning of the *Rolling Stock Circulation* phase, is considered in detail.

In the *Train-Unit Assignment Problem* (TUAV), we are given a set of *timetabled trips*, each with a required number of passenger seats, and a set of different *train units*, each consisting of a self-contained train with an engine and a set of wagons, and having a cost and a given number of available seats. TUAV calls for the minimum cost assignment of the train units to the trips, possibly combining more than one train unit for a given trip, so as to fulfil the seat requests.

Two *Integer Linear Programming* (ILP) formulations of TUAV are presented, and effective procedures are introduced for solving the corresponding Linear Programming (LP) relaxation. Additional relaxations, based on the Lagrangian approach and on the solution of a restricted problem associated with a peak period (i.e., with a subset of simultaneous trips that must be assigned to distinct train units), are considered for the computation of tight lower bounds. Constructive heuristic algorithms, based on the previously considered relaxations, are proposed, and their solutions are improved by applying local search procedures.

Extensive computational results on real-world instances are reported, showing the effectiveness of the proposed bounding procedures and heuristic algorithms.